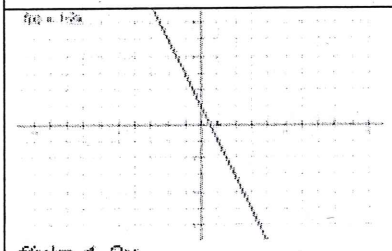
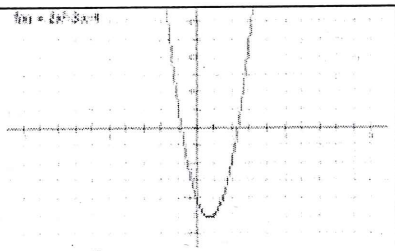
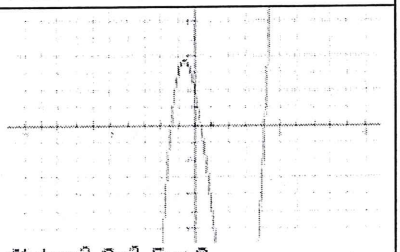
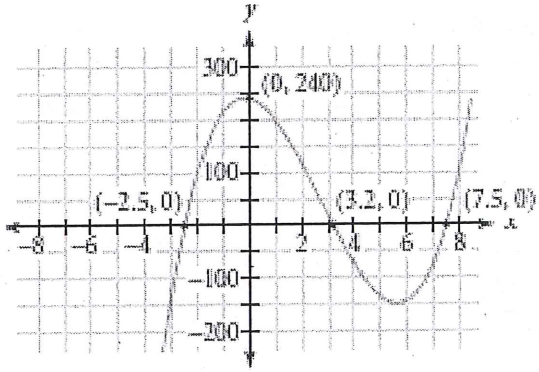
7.6 – Factoring Polynomials

Daily Objectives:

1. Explore functions defined by 3rd degree polynomials.
2. Use graphs of polynomial equations to find the roots and write the equations in factored form.
3. Relate the graphs of polynomial equations to the number and type of roots.
4. Identify possible degrees of a polynomial function by looking at its graph.

Linear	Quadratic	Cubic
$f(x)= a+bx$	$y=ax^2+bx+c$ General form $y=a(x-h)^2+k$ Vertex form $y=a(x-r_1)(x-r_2)$ Factored form	$f(x)=ax^3+bx^2+cx+d$ General Form $f(x)=a(x-r_1)(x-r_2)(x-r_3)$ Factored Form
 $f(x)= 1-2x$	 $f(x)=2x^2-3x-4$	 $f(x)=x^3-3x^2-5x+2$
Number or real roots? 1	2	3

Example 1: Write cubic functions for the graphs below:



$y=a(x+2.5)(x-3.2)(x-7.5)$

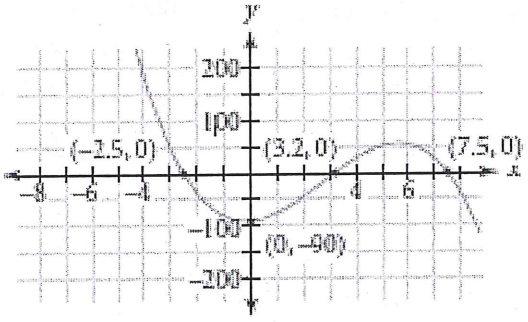
$240=a(0+2.5)(0-3.2)(0-7.5)$

$240=(2.5)(-3.2)(-7.5)a$

$\frac{240}{60}=\frac{60a}{60}$

$4=a$

$y=4(x+2.5)(x-3.2)(x-7.5)$



$y=a(x+2.5)(x-3.2)(x-7.5)$

$-90=a(0+2.5)(0-3.2)(0-7.5)$

$-90=(2.5)(-3.2)(-7.5)a$

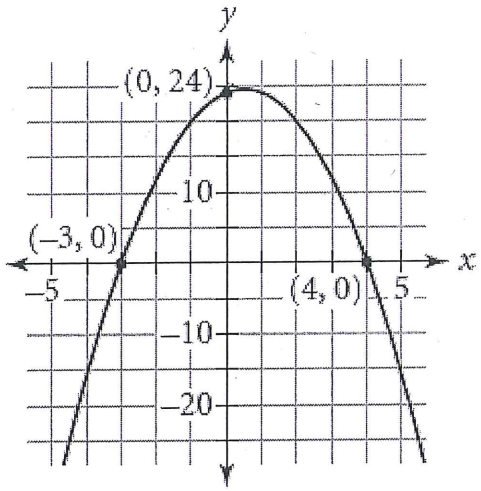
$\frac{-90}{60}=\frac{60a}{60}$

$-1.5=a$

$y=-1.5(x+2.5)(x-3.2)(x-7.5)$

Example 2: Write the factored form of the quadratic function for each graph. *Don't forget the vertical scale factor*.

a.



$y = a(x+3)(x-4)$

$24 = a(0+3)(0-4)$

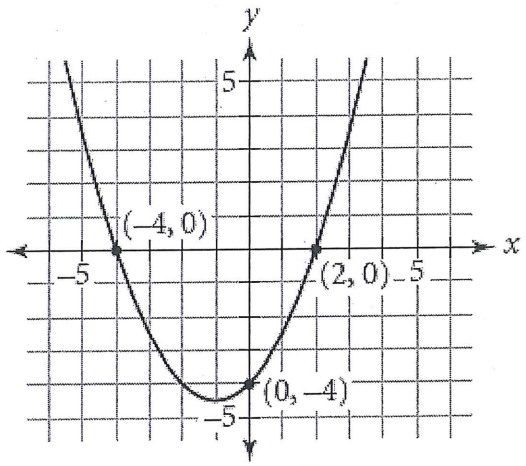
$24 = 3(-4)(a)$

$\frac{24}{-12} = \frac{-12a}{-12}$

$-2 = a$

$y = -2(x+3)(x-4)$

b.



$y = a(x+4)(x-2)$

$-4 = a(0+4)(0-2)$

$-4 = 4(-2)a$

$\frac{-4}{-8} = \frac{-8a}{-8}$

$\frac{1}{2} = a$

$y = \frac{1}{2}(x+3)(x-4)$

Example 3: Use the graph of each function to determine its factored form.

a. $y = x^2 - x - 2$

$y = (x-2)(x+1)$

Zeros: $x = 2$ $x = -1$

b. $y = 4x^3 + 8x^2 - 36x - 72$

$(0, -72)$

Zeros: $x = -3$ $x = -2$ $x = 3$

$y = a(x+3)(x-3)(x+2)$

$-72 = a(3)(-3)(2)$

$\frac{-72}{-18} = \frac{-18a}{-18}$

$4 = a$

$y = 4(x+3)(x-3)(x+2)$

c. $y = 3x^3 + 3x^2 - 30x + 24$

Zeros: $x = -4$ $x = 1$ $x = 2$ $(0, 24)$

$y = a(x+4)(x-1)(x-2)$

$24 = a(0+4)(0-1)(0-2)$

$\frac{24}{8} = \frac{8a}{8}$

$3 = a$

$y = 3(x+4)(x-1)(x-2)$

Example 4: Convert each polynomial function to general form.

a. $y = -2(x-2.5)(x+2.5)$

$-2[x^2 - 2.5x + 2.5x - 6.25]$

$-2[x^2 - 6.25]$

$y = -2x^2 + 12.5$

b. $y = -0.5(x+3)^2$

$= -.5(x+3)(x+3)$

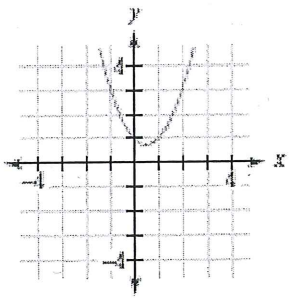
$= -.5[x^2 + 3x + 3x + 9]$

$= -.5[x^2 + 6x + 9]$

$y = -.5x^2 + 3x + 4.5$

Example 5: Discuss what each graph tells you about the factors of the function:

$y = x^2 - x + 1$



No real zeros –
Cannot be factored
only complex zeros

$$\frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2}$$

$$\frac{1 \pm \sqrt{1-4}}{2}$$

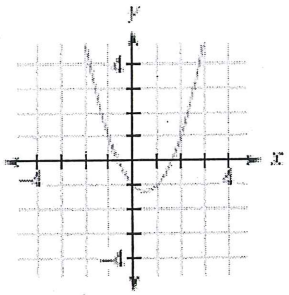
$$\frac{1 \pm \sqrt{-3}}{2}$$

$$\frac{1 \pm \sqrt{3}\sqrt{-1}}{2}$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

$$y = \left(x - \left(\frac{1 + i\sqrt{3}}{2}\right)\right)\left(x - \left(\frac{1 - i\sqrt{3}}{2}\right)\right)$$

$y = x^2 - x - 1$



Two real zeros
Two factors

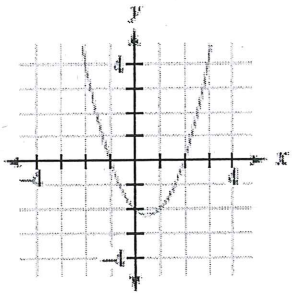
$$\frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2}$$

$$\frac{1 \pm \sqrt{1+4}}{2}$$

$$x = \frac{1 \pm \sqrt{5}}{2}$$

$$y = \left(x - \left(\frac{1 + \sqrt{5}}{2}\right)\right)\left(x - \left(\frac{1 - \sqrt{5}}{2}\right)\right)$$

$y = x^2 - x - 2$



Two real zeros
Two factors

$$y = (x-2)(x+1)$$